General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\tan^{-1} x = y$, then $\frac{dy}{dx}$ is equal to :

(A) $(\sec^{-1} x)^2$ (B) $\sec^2 y$

(C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\cos^2 y$

2. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is :

(A) 400π cm^3/cm (B) 100π cm^3/cm

(C) 50π cm^3/cm (D) 25π cm^3/cm

3. $\int \frac{dx}{\sqrt{e^{-2x}-1}}$ is equal to :

(A) $\sin^{-1} e^{-x} + C$ (B) $\log|e^{-x} + \sqrt{e^{-2x}-1}| + C$

(C) $\sin^{-1} e^{x} + C$ (D) $\log|e^{-x} - \sqrt{e^{-2x}-1}| + C$

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4. $\int_{-1}^{1} (1 - |x|)\, dx$ is equal to :

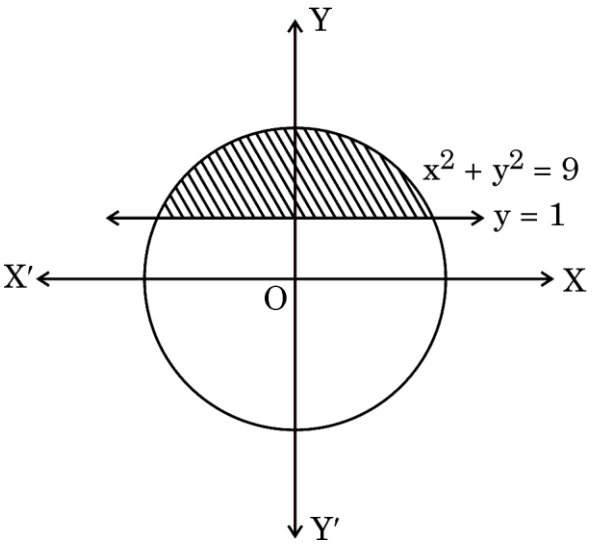
(A) $2\int_{0}^{1} (1 + x)\, dx$

(B) $2\int_{-1}^{0} (1 + x)\, dx$

(C) 0

(D) $2\int_{-1}^{0} (1 - x)\, dx$

5. The area of the shaded region of the circle given below is equal to :



(A) $\int_{1}^{3} \sqrt{9 - y^2}\, dy$

(B) $2\int_{1}^{3} \sqrt{9 - y^2}\, dy$

(C) $\int_{0}^{3} \sqrt{9 - x^2}\, dx$

(D) $2\int_{0}^{3} \sqrt{9 - x^2}\, dx$

6. $\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ?

(i) $F(x, y) = 3x + 2y$

(ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$

(iii) $F(x, y) = e^{y/x} + 1$

(iv) $F(x, y) = \sqrt{x^2 + y^2} - y$

(A) (i) and (ii)

(B) (i), (ii) and (iii)

(C) $\frac{5}{3}$ (D) 7

7. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \leq |\vec{a}| \, |\vec{b}|$ (B) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$

(C) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$ (D) $|\vec{a} \times \vec{b}| \geq |\vec{a}| \, |\vec{b}|$

8. If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10 |\vec{b}|$, then :

(A) $|\vec{a}| = \sqrt{2}$ (B) $|\vec{b}| = \sqrt{2}$

(C) $|\vec{b}| = 10\sqrt{2}$ (D) $|\vec{a}| = \frac{10}{\sqrt{2}}$

9. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then :

(A) $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ (B) $\sin\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(C) $\tan\theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$ (D) $\cos\theta = | l_1 l_2 + m_1 m_2 + n_1 n_2 |$

10. Direction ratios of lines l_1 and l_2 respectively are <1, – 2, 3> and <–2, p, – 6>. The value of p for which $l_1 \parallel l_2$, is :

(A) – 4 (B) 4

(C) – 10 (D) 10

11. If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E|F) - P(F|E)$ is equal to :

(A) $\frac{2}{7}$ (B) $\frac{3}{35}$

(C) $\frac{1}{70}$ (D) $\frac{1}{7}$

12. The domain of $\sin^{-1}(1 - 2x)$ is :

(A) [–1, 1] (B) [–1, 3]

(C) [– 2, 2] (D) [0, 1]

13. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :

(A) – 1 (B) 1

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{2^x + 1}\, dx$$

29. A survey was conducted on the patients who have undergone knee replacement surgeries.

It was found that, Robotic Knee replacement surgeries have 90% success rate.

On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that :

(i) exactly one surgery is successful,

(ii) at most two surgeries are successful.

30. (a) Find :

$$\int \frac{dx}{x^{1/2} + x^{1/3}}$$

OR

(b) Find :

$$\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$$

31. Evaluate :

(iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ? *2*