General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

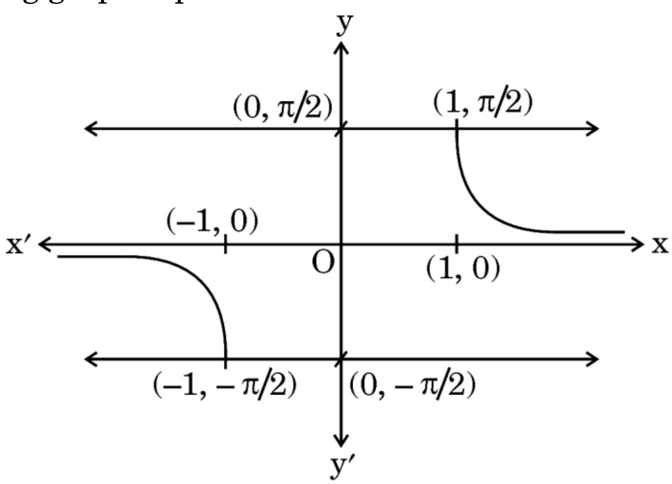
(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The following graph represents :



(A) $y = \cos^{-1} x$ (B) $y = \sec^{-1} x$

(C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$

2. If $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \dfrac{|i - 3j|}{2}$, then A' is :

(A) $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{5}{2} & 1 \end{bmatrix}$

3. The principal value of $\sec^{-1}\left(\sqrt{2}\right) + 2\ \text{cosec}^{-1}\left(-\sqrt{2}\right)$ is :

(A) $-\frac{\pi}{2}$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

4. If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is :

(A) $\frac{4}{7}$

(B) $-\frac{3}{7}$

(C) 4

(D) -4

5. Differential of e^{e^x} with respect to x is :

(A) $\log x$

(B) e^{e^x}

(C) $e^x e^{e^x}$

(D) $(e^x)^2$

6. The surface area of a sphere when its volume changes at the same rate as its radius is :

(A) 4π sq. units

(B) 1 sq. unit

(C) 4 sq. units

(D) π sq. units

7. If $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then the value of k is :

(A) 0 (B) – 2

(C) –1 (D) 2

8. The greatest integer function, f(x) = [x], 0 < x < 3 is ***not*** differentiable at how many points ?

(A) At only one point (B) At only two points

(C) At no point (D) At three points

9. $\int \frac{dx}{\sqrt{25 - 16x^2}}$ is equal to :

(A) $\frac{1}{5} \sin^{-1} 4x + C$ (B) $\frac{1}{25} \sin^{-1} 16x + C$

(C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + C$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + C$

10. If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is :

(A) e (B) $\frac{\pi}{4}$

(C) 0 (D) $-\frac{\pi}{4}$

11. The area of the region bounded by the curve y = x and x-axis, between x = 0 and x = 2 is :

(A) 2 sq. units (B) $\frac{1}{2}$ sq. unit

(C) 1 sq. unit (D) 4 sq. units

12. Product of the order and degree of differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \lambda \left(\frac{d^3y}{dx^3}\right)^2$ is :

(A) 5 (B) 6

(C) 2 (D) 3

13. Which of the following is ***not*** a Linear Differential Equation ?

(A) $(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$

(B) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

(C) $x(1 + y^2)\, dx - y(1 + x^2)\, dy = 0$

(D) $y\, dx - (x + 3y^2)\, dy = 0$

14. The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is :

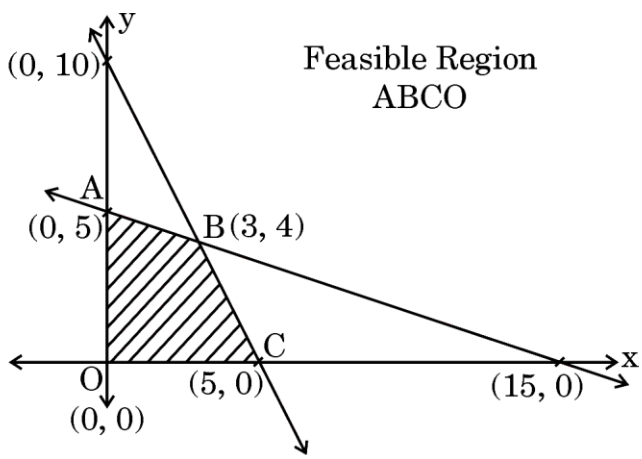
(A) unbounded in 1^{st} quadrant

(B) bounded in 1^{st} quadrant

(C) unbounded in 2^{nd} quadrant

(D) bounded in 2^{nd} quadrant

15. In the graph, the feasible region representing the Linear Programming Problem for maximising objective function $Z = px + qy$, $p, q > 0$ is shaded. If all points on segment AB give max (Z), then which of the following is true ?



(A) $p = 2q$

(B) $p = 3q$

(C) $q = 3p$

(D) $q = 2p$

16. If $(3\hat{i} - 2\hat{j} + 5\hat{k}) \times (4\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are :

(A) $p = -\frac{2}{3}, q = \frac{5}{3}$

(B) $p = -\frac{8}{3}, q = \frac{20}{3}$

(C) $p = \frac{20}{3}, q = -\frac{8}{3}$

(D) $p = 0, q = 0$

17. Three points A(0, 1, 1), B(2, 0, –1) and C(1, 0, 3) form Δ ABC. The ar (Δ ABC) is :

(A) $\frac{\sqrt{53}}{2}$ sq. units (B) $\sqrt{53}$ sq. units

(C) $\frac{\sqrt{11}}{2}$ sq. units (D) $\sqrt{11}$ sq. units

18. A box contains 4 red, 5 blue and 1 green marble. A child randomly takes out a marble from the box, notes down the colour and puts it back in the box. If the activity is repeated 3 times, what is the probability that at least one marble is red ?

(A) $\frac{27}{125}$ (B) $\frac{8}{125}$

(C) $\frac{2}{125}$ (D) $\frac{98}{125}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If A and B are two square matrices such that AB and BA are defined, then it is not necessary that AB = BA.

Reason (R) : Product of two diagonal matrices of same order is commutative.

20. *Assertion (A) :* A function $f : N \rightarrow N$ given by $f(x) = x^3 + 2, \forall x \in N$ is one-one but not onto.

Reason (R) : Since $\forall y \in N$ (Codomain), there does not exist $x = (y - 2)^{1/3}$ in N (Domain) such that $f(x) = x^3 + 2 = y$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Evaluate :

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$$

22. (a) Show that the function $f(x) = \begin{cases} \dfrac{\cos x}{-x+\frac{\pi}{2}}, & x \neq \dfrac{\pi}{2} \\ 1, & x = \dfrac{\pi}{2} \end{cases}$

is continuous at $x = \dfrac{\pi}{2}$.

OR

(b) Find whether the function $f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.

23. Find the values of x for which $f(x) = x^x,\ x > 0$ is increasing.

24. If the position vectors of three points A, B and C are $3\hat{i} + \hat{j}$, $5\hat{i} + 6\hat{j} - 3\hat{k}$ and $4\hat{j}$ respectively, then show that they form an isosceles triangle.

25. (a) Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.

OR

(b) A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \to B$ is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.

OR

(b) Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x, y) : (x - y) \text{ is divisible by } n,\ x, y \in Z\}$. Determine if R is an equivalence relation.

27. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then compute $A^2 - 7A + 10\,I$.

28. (a) If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.

OR

(b) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.

29. Solve the following Linear Programming Problem graphically :

Maximise $Z = 200x + 120y$

subject to the constraints

$$x + y \leq 300$$

$$3x + y \leq 600$$

$$x - y \geq -100$$

$$x, y \geq 0$$

30. (a) Find a point on the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point (1, 2, 3).

OR

(b) Find the shortest distance between the lines

$$\vec{r} = (4+\lambda)\hat{i} + (2\lambda - 1)\hat{j} - 3\lambda\hat{k}$$

$$\vec{r} = (1+2\mu)\hat{i} + (2-5\mu)\hat{k} + (4\mu - 1)\hat{j}.$$

31. In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x}{(x-1)(x^2+4)} dx$$

OR

(b) Evaluate :

$$\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$$

33. Using integration, find the area of the region enclosed by the curve $y = |x - 6|$, the x-axis, and between $x = 4$ and $x = 8$.

34. (a) Solve the differential equation $y\, e^y\, dx = (y^3 + 2x\, e^y)\, dy$, when $y(0) = 1$.

OR

(b) Find the general solution of the differential equation

$$(x^3 - 3xy^2)\, dx = (y^3 - 3x^2y)\, dy.$$

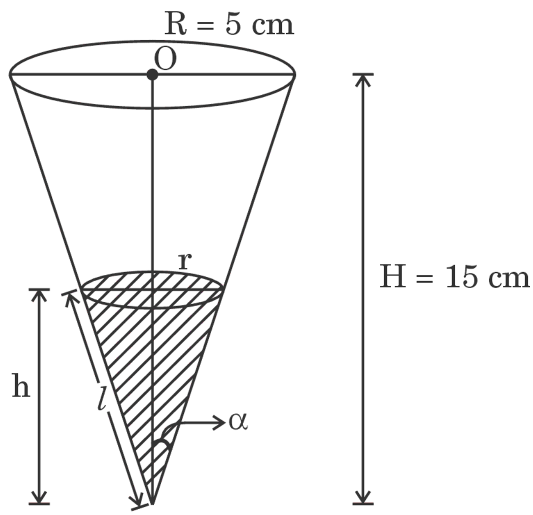
35. Find the equation of a line (in vector and cartesian form) that passes through the point of intersection of lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. At a birthday party, children are being served orange juice in conical cups, as shown in the figure.



Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0{\cdot}1\ cm^3/s$.

On the basis of the above information, answer the following questions :

(i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α. *1*

(ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? *1*

(iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? *2*

OR

(iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing. *2*

Case Study – 2

37. A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.

On the basis of the above information, answer the following questions :

(i) Write the equations representing the various dimensions and express them as the matrix equation AX = B. *1*

(ii) Find if A^{-1} exists. Justify your answer. *1*

(iii) (a) Find A^{-1}. *2*

OR

(iii) (b) Find $A^2 + 7I$. *2*

Case Study – 3

38. An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, and Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where i = 1, 2, 3.

Based on the above information, answer the following questions :

A person selects a cap.

(i) What is the probability that he selects a red cap ? *2*

(ii) If he selects a green cap, what is the probability that the cap has come from Box II ? *2*