General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises 20 multiple choice questions (MCQs) of 1 mark each.

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as $f(x) = x^3$, then f is :

(A) one-one onto function

(B) many-one function

(C) one-one but not onto function

(D) neither one-one nor onto function

2. The number of all possible matrices of order 2×3 with each entry 1 or 2 is :

(A) 16 (B) 81

(C) 64 (D) 32

3. If $A = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$, then $A + A' = I$, if α is equal to :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$

4. If the area of a triangle is 15 sq. units with vertices (3, – 7), (6, 3) and (k, 3), then the value(s) of k is/are :

(A) 3 (B) 9

(C) – 9, – 3 (D) 9, 3

5. The interval in which $y = x^3 . e^{-3x}$ is an increasing function is :

(A) $(-1, 2)$ (B) $(-\infty, 2)$

(C) $(1, \infty)$ (D) $(0, 1)$

6. The value of k for which $f(x) = \begin{cases} \frac{\sin 7x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

is continuous at x = 0 is :

(A) $\frac{5}{7}$ (B) $\frac{7}{5}$

(C) 0 (D) $\frac{1}{5}$

7. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is equal to :

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

8. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to :

(A) $\cot x + \tan x + C$ (B) $-\cot x + \sec x + C$

(C) $-\cot x + \tan x + C$ (D) $-(\tan x + \cot x) + C$

9. $\int \frac{dx}{x^2 + 6x + 10}$ is equal to :

(A) $x \tan^{-1}(x + 3) + C$ (B) $\tan^{-1}(x + 3) + C$

(C) $\frac{1}{3} \tan^{-1}(x + 3) + C$ (D) $\tan^{-1} x + C$

10. The general solution of the differential equation $\frac{dy}{dx} = e^{y-x}$ is :

(A) $e^x - e^y = C$ (B) $e^{-y} + e^x = C$

(C) $e^{-y} - e^{-x} = C$ (D) $e^{-x} + e^{-y} = C$

11. The integrating factor of $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$ is :

(A) $\tan y$ (B) $\tan^{-1} y$

(C) $e^{\tan^{-1} y}$ (D) $e^{\tan^{-1} x}$

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + 2\vec{b}$ is a unit vector if :

(A) $\theta = \frac{\pi}{2}$ (B) $\theta = \frac{2\pi}{3}$

(C) $\theta = \frac{3\pi}{4}$ (D) $\theta = \pi$

13. The area (in square units) of a parallelogram, whose diagonals are $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is :

(A) $2\sqrt{14}$ (B) $\sqrt{14}$

(C) $\sqrt{38}$ (D) $2\sqrt{6}$

14. The corner points of the feasible region for an LPP are (0, 2), (6, 0), (6, 8) and (0, 5). Let Z = 4x + 6y be the objective function. (Maximum of Z) – (Minimum of Z) is equal to :

(A) 60 (B) 48

(C) 42 (D) 18

❀

15. In an LPP, if the objective function has the same minimum value on two corner points of the feasible region, then the number of points at which the minimum value of objective function occurs is :

(A) 0 (B) 2

(C) Infinite (D) Finite

16. If events A and B are such that $P(A|B) = P(B|A)$, then :

(A) $A \subset B$ but $A \neq B$ (B) $A = B$

(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

17. The probability of getting an odd prime number on each die, when a pair of dice is rolled, is :

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$

(C) $\frac{1}{36}$ (D) $\frac{1}{6}$

18. In a family with three children, the probability that the eldest child is a boy, given that the family has at least one boy, is :

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4}{7}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $f(x) = e^{-x}$ is a decreasing function, where $x \in \mathbb{R}$.

Reason (R) : If $f'(x) < 0$, then $f(x)$ is a decreasing function.

20. *Assertion (A) :* For two independent events A and B, $P(A|B) = P(A)$.

Reason (R) : A and B are independent events, then $P(A \cap B) = P(A) . P(B)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) For the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, find the numbers a and b so that $A^2 + aA + bI = O$.

OR

(b) Find the equation of line joining the points $(-1, -1)$ and $(1, 3)$, using determinants.

22. (a) The volume of a spherical balloon is increasing at the rate of 20 cm^3/s. Find the rate of change of its surface area when its radius is 4 cm.

OR

(b) Find the intervals on which the function $f(x) = x^3 + 2x^2 - 1$ is strictly increasing.

23. Find :

$$\int \frac{(x-4)\, e^x}{(x-2)^3}\, dx$$

24. Find a unit vector perpendicular to both of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$, where A, B and C are $(3, -1, 2)$, $(1, -1, -3)$ and $(4, -3, 1)$, respectively.

25. Find the image of the point $(2, -1, 5)$ in the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Find the value of

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right).$$

OR

(b) Prove that the function $f : N \to N$, $f(n) = (n^2 + n + 1)$ is one-one but not onto.

27. (a) Find :

$$\int \frac{x}{\sqrt{7-6x-x^2}}\,dx$$

OR

(b) Find :

$$\int \frac{2}{(1-x)(1+x^2)}\,dx$$

28. (a) Solve the differential equation :

$$x\frac{dy}{dx} = y - x\tan\frac{y}{x}$$

OR

(b) Solve the differential equation :

$$\frac{dy}{dx} = e^{x+y} + e^{x-y}$$

❀

29. Evaluate :

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

30. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

31. The corner points of the feasible region determined by the system of linear constraints in an LPP are (0, 10), (5, 5), (15, 15) and (0, 20). Let Z = px + qy, p, q > 0, be the objective function, then find the relation between p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20).

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations :

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

OR

(b) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, then show that $(AB)^{-1} = B^{-1}A^{-1}$.

33. (a) Of all the closed right circular cylindrical cans of given volume 200 cm^3, find the dimensions of the can which has the minimum surface area.

OR

(b) Show that the height of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

34. Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

35. Find the shortest distance between the lines :

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$, and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

SECTION E

This section comprises 3 case-study based questions of 4 marks each.

Case Study – 1

36. Two friends Ramesh and Divya have started playing Ludo at their house. While rolling the die, they observed each time, that the possible outcomes are {1, 2, 3, 4, 5, 6}. Let A be the set of players and B be the set of all possible outcomes, so A = {Ramesh, Divya} and B = {1, 2, 3, 4, 5, 6}.

Based on the above information, answer the following questions :

Let relation $R : B \to B$ be defined as

$R = \{(x, y) : y \text{ is divisible by } x\}$.

(i) Write all elements of R. *1*

(ii) Find whether R is reflexive or not. *1*

(iii) (a) Check whether R in B is symmetric or transitive. *2*

OR

(iii) (b) Let $R_1 : B \to B$ is defined as

$R_1 = \{(x, y) : x = 2y, x, y \in B\}$.

Check whether R_1 is reflexive, symmetric or transitive. *2*

Case Study – 2

37. A Math teacher of a school is teaching the derivative of function in parametric form, to her students.

She explains that sometimes the relation between two variables, (say x and y), is neither explicit nor implicit, but the two variables are expressed in a third variable, (say t). So we have x = f(t) and y = g(t). Such form is called parametric form. For finding the derivative in such case we use the chain rule as

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Based on the above information, answer the following questions :

(i) If $x = at^3$, $y = at^5$, then find $\frac{dy}{dx}$. *1*

(ii) If $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$, then find $\frac{dy}{dx}$. *1*

(iii) (a) If x = 2 sin t + sin 2t and y = 2 cos t + cos 2t, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$. *2*

OR

(iii) (b) If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$, then find $\frac{dy}{dx}$. *2*

Case Study – 3

38. Two bags contain balls such that Bag I contains 5 red and 6 black balls, while Bag II contains 3 red and 4 black balls.

Based on the above information, answer the following questions :

(i) One ball is drawn at random from each of the two bags. Find the probability of getting both red balls. *1*

(ii) One ball is drawn at random from each of the two bags. Find the probability of getting one red and one black ball. *1*

(iii) (a) One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. Find the probability of getting a red ball. *2*

OR

(iii) (b) One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. Find the probability of getting a black ball. *2*