BIHAR BOARD – 2018

Maths

Time: 3hrs 15minutes **Max Marks: 100**

1. $0.3\bar{2}$ can be expressed in the form of $\frac{p}{q}$ (p, q are interger, q≠0) as -

(a) $\frac{8}{25}$ (b) $\frac{29}{90}$ (c) $\frac{32}{99}$ (d) $\frac{32}{199}$

Ans. $\frac{29}{90}$

2. An irrational number between 2 and 2.5 is –

(a) √11 (b) √5 (c)√22.5 (d)√12.5

Ans. √5

3. Which of the following statement is true?

a) Product of two irrational numbers is always irrational.
b) Product of a rational number and an irrational numbers is always irrational.
c) Sum of two irrational numbers can never be irrational.
d) Sum of an integer and a rational number can never be an integer.

Ans. Product of a rational number and an irrational numbers is always irrational.

4. If n is a natural number then √n is-

(a) Always a natural number (b) Always an irrational number

(c) Always a rational number

(d) Sometimes a natural number and sometimes an irrational number

Ans. Sometimes a natural number and sometimes an irrational number

5. If N is a sum of first 13986 prime numbers, then N is always divisible by –

(a) 6 (b) 4 (c) 8 (d) None of these

Ans. None of these

6. Degree of polynomial $y^3 - 2y^2 - \sqrt{3}y + \frac{1}{2}$ is-

(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) $\frac{3}{2}$

Ans. 3

7. The sum and product of the zeros of a quadratic polynomial are 2 and -15 respectively. The quadratic polynomial is –

(a) $x^2 - 2x + 15$ (b) $x^2 - 2x - 15$ (c) $x^2 + 2x - 15$ (d) $x^2 + 2x + 15$

Ans. $x^2 - 2x - 15$

8. For what value of k ,the pair of linear equation $2x - y - 3 = 0, 2kx + 7y - 5 = 0$ has a unique solution x=1and y =-1?

(a) 3 (b) 4 (c) 6 (d) -6

Ans. 6

9. If one of the zeroes of cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, then the product of other two zeros is-

(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

Ans. $\frac{c}{a}$

10. The value of p for which the polynomial $x^3 + 4x^2 - px + 8$ is exactly divisible by (x-2) is-

(a) 0 (b) 3 (c) 5 (d) 16

Ans. 16

11. Which of the following is the solution of x-2y =0 and 3x+4y = 10?

(a) x = 2, y =1 (b) x = 1, y = 1 (c) x = 2, y = 2(d) x =3, y =1

Ans. x = 2, y =1

12. If $10^{2y} = 25$, then 10^{-y} equals-

(a) $\frac{1}{5}$ (b) $\frac{1}{50}$ (c) $\frac{1}{625}$ (d) $\frac{(-)1}{5}$

Ans. $\frac{1}{5}$

13. If (2k-1, k) is solution of the equation 10x-9y =12,then k =......

(a) 1 (b) 2 (c) 3 (d) 4

Ans. 2

14. If $x = \sqrt{7 + 4\sqrt{3}}$ then $x + \frac{1}{x}$ is-

(a) 4 (b) 3 (c) 2 (d) 6

Ans. 4

15. If (x+1) is a factor of $(x^n + 1)$ then n is necessarily-

(a) An odd integer (b) An even integer

(c) A negative integer (d) A positive integer

Ans. An odd integer

16. If √3tanθ = 3sinθ, then $sin^2\theta - cos^2\theta$ is equals to-

(a) √3 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$

Ans. $\frac{1}{3}$

17. $\frac{2\tan 30^0}{1 + \tan^2 30^0}$ is equals to-

(a) sin 60° (b) cos 60° (c) tan 60° (d) sec 60°

Ans. sin 60°

18. If sec A = cosec B = $\frac{13}{12}$, then (A+B) is equal to-

(a) Zero (b)>90° (c) 90° (d) <90°

Ans. 90°

19. If in a triangle ABC, ∠A and ∠B are complementary, then cot C is –

(a) $\frac{1}{\sqrt{3}}$ (b) 0 (c) 1 (d)$\sqrt{3}$

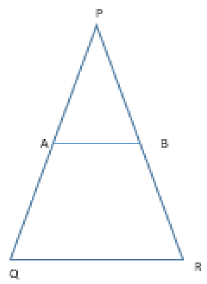
Ans. 0

20. If tan (α + β) =$\sqrt{3}$ and tan α = $\frac{1}{\sqrt{3}}$, then tan β =

(a) $\frac{1}{6}$ (b) $\frac{1}{7}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{7}{6}$

Ans. $\frac{1}{\sqrt{3}}$

21. If the sun's ray inclination increases from 45° to 60°, the length of shadow of a tower decreases by 50m. Height of tower (in m) is –



(a) 50(√3-1)(b) 75(3-√3) (c) 25(√3+1) (d) 25(3+√3)

Ans. 25(√3+1)

22. The angle of elevation of the sun, when length of shadow of a vertical pole is equal to its height, is –

(a) 30° (b) 45° (c) 60° (d) 90°

Ans. 45°

23. If two towers of height h_1 *and* h_2 subtend angle 60° and 30° respectively at the mid- point of line joining their bases, then $\frac{h_1}{h_2}$ is-

(a) 3:1 (b) 1:2 (c) √3:1 (d) 1:√3

Ans: 3:1

24. A tree of 6m height casts a 4m long shadow. At the same time, a flag post casts a shadow 50m long. The height of post (in m) is –

(a) 75 (b) 25 (c) 15 (d) 10

Ans. 75

25. A pole 6m high casts a shadow 2√3 m long on ground, the sun's elevation is –

(a) 60° (b) 45° (c) 30° (d) 15°

Ans. 60°

26. The line x = 2 and y = -2 are –

(a) Perpendicular to each other

(b) Parallel to each other

(c) Neither parallel nor perpendicular to each other

(d) Nothing can be said

Ans. Parallel to each other

27. The area of the triangle formed by point A(0, 1), B(0, 5) and C(3, 4) is (in sq. Units) –

(a) 16 (b) 8 (c) 6 (d) 4

Ans. 6

28. The area of triangle whose vertices are (-4, 0), (0, 3) and (0, 0)is –

(a) 36 (b) 12 (c) 6 (d) 1

Ans. 6

29. The coordinates of the point which divides the line segment joining the points (1, 1) and (2 ,3)in the ratio 2:3 are –

(a) $\frac{7}{5},\frac{9}{5}$ (b) (7, 9) (c) $\frac{7}{3},3$ (d) None of these

Ans. $\frac{7}{5},\frac{9}{5}$

30. The distance of point (-3 , 4) form the origin is –

(a) 3 (b) -3 (c) 4 (d) 5

Ans. 5

31. In the triangle PQR, AB || QR . The ratio of area of two similar ΔPAB & ΔPQR is 1:2, then $\frac{PQ}{AQ} =$

(a) √2:1 (b) 1:(√2-1) (c) 1:(√2+1) (d) None of these

Ans. None of these

32. In the following figure, O is the centre of the circle. $\angle BAC = 60°$, then

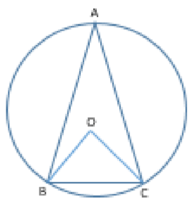
$\angle OBC =$

(a) 120° (b) 60° (c) 40° (d) 30°

Ans. 60°

33. PT and PS are the tangents to the circle with centre O. If $\angle TPS = 65°$ then

$\angle OTS =$



(a)32° (b) 32.5° (c) 45° (d) 57.5°

Ans. 57.5°

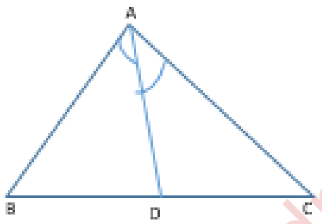
34. To divide the line segment AB in the ratio 2: 3, first a ray AX is drawn so that $\angle$BAX is an acute angle and then at equal distances A_1, A_2...... are marked on the ray. For such ratio, minimum number of these points to be selected, is –

(a) 3 (b) 5 (c) 8 (d) 6

Ans. 5

35. In ΔABC, AD is the internal bisector of $\angle A$. If BD = 5cm and BC = 7.5cm,

then $\frac{AB}{AC} =$



(a) 1 (b) 2 (c) 0.8 (d) 0.6

Ans. 2

36. Two isosceles triangles have equal angles and their areas are in the ratio 16:25. The ratio of their corresponding heights is-

(a) 4:5 (b) 5:4 (c) 3:2 (d) 1:4

Ans. 4:5

37. Two tangents, drawn at the end points of diameter of a given circle are always-

(a) parallel (b) perpendicular (c) intersect each other

(d) none of these

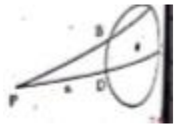
Ans. Parallel

38. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° , then $\angle$POA =

(a) 50° (b) 60° (c) 70° (d) 80°

Ans. 50°

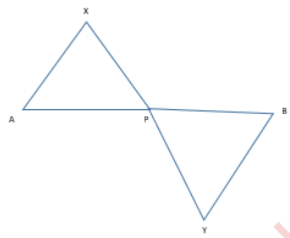
39. In the following figure, if PA = 8cm, PD = 4cm, CD = 3cm , then AB (in cm) is-



(a) 3 (b) 3.5 (c) 4 (d) 4.5

Ans. 4

40. In the figure, P divides AB internally in which ratio –



(a) 3:4 (b) 3:7 (c) 4:5 (d) None of these

Ans. 3:4

41. The height of a cylinder is 14cm and its curved surface area is 264 cm^2 . The volume of the cylinder (in cm^3) is –

(a) 296 (b) 369 (c) 396 (d) 503

Ans. 396

42. The ratio of the volumes of two spheres is 8:7. The ratio of their surface area is –

(a) 2:3 (b) 4:7 (c) 8:9 (d) 4:9

Ans. 4:9

43. If the curved surface area of a solid right circular cylinder of height h and radius r is one third of its surface area, then –

(a) h = $\frac{1}{3}r$ (b) h = $\frac{1}{2}r$ (c) h = r (d) h = 2r

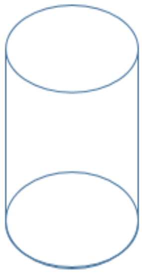
Ans. h = $\frac{1}{2}r$

44. The volume of air (in cm^3) displayed by sphere of diameter 12cm is –

(a) 144 (b) 144π (c) 288 (d) 288π

Ans. 288π

45. A cube of 5cm can be divided into how many number of cubes, each of 1cm side –



(a) 5 (b) 50 (c) 125 (d) 250

Ans. 125

46. If the median of 4 consecutive odd numbers is 6, then the largest number is

(a) 5 (b) 9 (c) 15 (d) 21

Ans. 9

47. The mode of 6, 4, 3, 6, 4, 3, 4, 6, 5 and x can be –

(a) only 5 (b) both 4 and 6 (c) both 3 and 6 (d) 3,4 or 6

Ans. both 4 and 6

48. Three unbiased coins are tossed. The probability of getting at least 2 heads is –

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Ans. $\frac{1}{2}$

49. In a simultaneous throw of two dice, the probability of getting a total of 7 is-

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Ans. $\frac{1}{6}$

50. Which of the following cannot be the probability of an event?

(a) 1.1 (b) 0.5 (c) 0.9 (d) 0.1

Ans. 1.1

Section B

1. Use Euclid's division algorithm to find HCF of 870 and 225.

Ans. Using Euclid's division algorithm, we get

$$870 = 225\times 3 + 195$$
$$225 = 195\times 1 + 30$$
$$195 = 30\times 6 + 15$$
$$30 = 15\times 2 + 0$$
$\therefore$ H.C.F.=15

2. Without actually performing the long division, state whether the rational number $\frac{13}{343}$ is terminating or non- terminating repeating decimal expansion.

Ans. Given term is $\frac{13}{343}$

Factorizing the denominator, we get

$343 = 7\times 7\times 7 = 7^3$, which is not in the form $2^m \times 5^n$.

Therefore, $\frac{13}{343}$ is non- terminating repeating decimal expansion.

3. If p and q are zeros of polynomial f(x) = $2x^2 - 7x + 3$ find the value of $p^2 + q^2$.

Ans. Given:

$$\text{f(x)} = 2x^2 - 7x + 3$$
$$= 2x^2 - 6x - x + 3$$
$$= 2x(x-3) - 1(x-3)$$
$$= (x-3)(2x-1)$$

So,

$$x = 3 \text{ or } \frac{1}{2}$$

we can write,

p = 3 and q = $\frac{1}{2}$

Therefore,

$$p^2 + q^2$$
$$= 3^2 + \left(\frac{1}{2}\right)^2$$
$$= 9 + \frac{1}{4}$$
$$= \frac{37}{4}$$

4. 10 students of class X took part in mathematics quiz. If number of girls is 4 more than number of boys, form a pair of linear equation for it.

Ans. Let the number of girls be x and the number of boys be y.

Given, total number of students = 10

$\therefore x + y = 10$

According to question,

Number of girls = 4more than number of boys

$$x = 4 + y$$
$$\therefore x - y = 4$$

Hence, the required pair of linear equation is $x - y = 4$ and $x + y = 10$.

5. Prove that $\sqrt{3}$ is an irrational number.

Ans. Let us assume that $\sqrt{3}$ is a rational number.

As we know a rational number should be in the form of $\frac{p}{q}$

where p and q are co- prime number.

So,

$\sqrt{3} = \frac{p}{q}$ { where p and q are co- prime)

$\Rightarrow p = \sqrt{3}q$

Now, by squaring both the side

we get,

$$p^2 = (\sqrt{3}q)^2$$
$$\Rightarrow p^2 = 3q^2 ....(i)$$

So,

If 3 is the factor of p^2

then, 3 is also a factor of p ..... (ii)

Now,

Let p = 3m {where m is any integer}

Squaring both sides,

$$p^2 = (3m)^2$$
$$\Rightarrow p^2 = 9m^2$$

putting the value of p^2 in equation (i)

we get,

$$9m^2 = 3q^2$$
$$\Rightarrow q^2 = \frac{9m^2}{3}$$
$$\Rightarrow q^2 = 3m^2$$

So,

If 3 is factor of q^2

then, 3 is also factor of q

Since

3 is factor of p & q both

So, our assumption that p & q are co- prime is wrong.

Hence, $\sqrt{3}$ is an irrational number.

6. Find the conditions which must be satisfied by the co-efficient of polynomial such that the sum of two zeros of the polynomial f(x) = $x^3 - px^2 + qx - r$ is zero.

Ans. Let the zeros of the given polynomial be α, β and δ.

Let the sum of α and β be zero.

Now,

Using the principle of sum of zeros and product of zeros,

$$\alpha + \beta + \delta = p$$
$$\Rightarrow \delta = p............(i)$$
$$And,$$
$$\alpha\beta + \beta\delta + \delta\alpha = q$$
$$\Rightarrow \alpha\beta + \delta(\alpha + \beta) = q$$
$$\Rightarrow \alpha\beta = q................(ii)$$

Also,

$$\alpha\beta\delta = r$$
$$\therefore pq = r \qquad \text{[from (i) and(ii)]}$$

Hence, the required condition is pq = r.

7. The angle of a triangle are x, y and 40°. The difference between two angles x and y is 30°. Find x and y.

Ans. Given, Angles are x, y and 40°.

x– y = 30

$$\Rightarrow x = 30 + y......(i)$$

We know,

Sum of angles of a triangle = 180

$$\Rightarrow x + y + 40 = 180$$
$$\Rightarrow 30 + y + y + 40 = 180 \qquad \text{[From (i)]}$$
$$\Rightarrow 2y = 180 - 70$$
$$\therefore y = \frac{110}{2} = 55^0$$

Putting value of y in (i),

$x = 30 + 55 = 85°$

8. Find the 11th term of the series 2, 4, 6, 8,......

Ans. Given,

First term (a) = 2

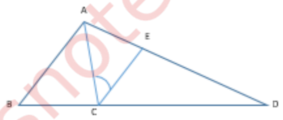
Common difference (d) =4-2= 2

We know, for nth term

$t_n = a + (n-1)d$

$$\begin{aligned} \therefore t_{11} &= 2 + (11-1)2 \\ &= 2 + 10 \times 2 \\ &= 2 + 20 \\ &= 22 \end{aligned}$$

9. In the given figure, $AC \perp BD$ and $CE \perp AD$, prove that $AC^2 = DA.AE$



Ans. We know,

$\Delta AED \sim \Delta EDC \sim \Delta ACD$

So,

$\frac{ar(\Delta AED)}{ar(\Delta ACD)} = \frac{AC^2}{AD^2}$............$(i)$

Here,

$$\frac{ar(\Delta AED)}{ar(\Delta ACD)} = \frac{\frac{1}{2}\times b\times h}{\frac{1}{2}\times b\times h}$$

$$= \frac{\frac{1}{2}\times AE\times CE}{\frac{1}{2}\times AD\times CE}$$

$$= \frac{AE}{AD}\ldots\ldots\ldots\ldots\ldots(ii)$$

Now, from (i) and (ii), we get

$$\frac{AC^2}{AD^2} = \frac{AE}{AD}$$

$$\Rightarrow AC^2 = \frac{AE\times AD^2}{AD}$$

$$\therefore AC^2 = AE\times AD$$

Hence , $AC^2 = DA.AE$ proved.

10. ΔABC and ΔDEF are similar. Areas of these two triangles are $9cm^2$ and $64cm^2$ respectively. If DE = 5.1cm, find AB.

Ans. Given, $\Delta ABC \sim \Delta DEF$, DE =5.1cm

Area of ΔABC and ΔDEF are $9cm^2$ and $64cm^2$ respectively.

Using similar triangle theorem, we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{9}{64} = \frac{AB^2}{(5.1)^2}$$

$$\Rightarrow AB^2 = \frac{9\times(5.1)^2}{64}$$

$$\Rightarrow AB = \sqrt{\frac{9\times(5.1)^2}{64}}$$

$$\therefore AB = \frac{3\times 5.1}{8} = 1.91cm$$

11. Find the length of a chord which is at a distance of 4cm from the centre of a circle of radius 6cm.

Ans. Given,

Distance of the chord from the centre (OR)=4cm

Radius of the circle(OP)=5cm

By Pythagoras theorem,

$$OP^2 = OR^2 + PR^2$$
$$\Rightarrow 5^2 = 4^2 + PR^2$$
$$\Rightarrow 25 = 16 + PR^2$$
$$\Rightarrow PR^2 = 9$$
$$\therefore PR = 3cm$$

Now,

Since, the perpendicular drawn from centre to chord bisects the chord.

So,

PQ = PR +QR = (3+3)= 6cm

Therefore, length of the chord is 6 cm.

12. Show that equal chords of a circle subtend equal angle at the centre.

Ans. Given: AB and CD are equal chords of the same circle with center as O.

To prove: angle AOB = angle COD

Proof: In triangle AOB and triangle COD.

AO = CO (radii of the same circle)

AB = CD (given)

OB = OC (radii of the same circle)

Therefore triangle AOB is congruent to triangle COD by SAS congruence rule..

This implies angle AOB is equal to angle COD.

Hence , equal chords of a circle subtend equal angle at the centre.

13. If $\tan\theta = \frac{3}{4}$, find $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

Ans. Given that ,

$\tan\theta = \frac{3}{4}$

<u>We know that</u>

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Suppose x, then

$\sin\theta = 3x$

$\cos\theta = 4x$

Now, putting the value in $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

$$= \frac{5\times 3x - 3\times 4x}{5\times 3x + 3\times 4x}$$
$$= \frac{15x - 12x}{15x + 12x}$$
$$= \frac{3x}{27x}$$
$$= \frac{1}{9}$$

14. If $\tan\theta + \sec\theta = l$, then prove that $\sec\theta = \frac{l^2+1}{2l}$

Ans. Given,

$\tan\theta + \sec\theta = l$

$\Rightarrow \tan\theta = l - \sec\theta$

$\Rightarrow \tan^2\theta = (l - \sec\theta)^2$ {squaring both sides}

$\Rightarrow \tan^2\theta = l^2 - 2l.\sec\theta + \sec^2\theta$

$\Rightarrow \sec^2\theta - 1 = l^2 - 2l.\sec\theta + \sec^2\theta$ $[\because \tan^2\theta + \sec^2\theta = 1]$

$\Rightarrow 2l.\sec\theta = l^2 + 1$

$\therefore \sec\theta = \frac{l^2+1}{2l}$

Hence proved.

15. If tan A = 1 and sin B = $\frac{1}{\sqrt{2}}$. Find the value of cos (A +B) , where A and B are acute angles.

Ans. Given,

$$\tan A = 1$$
$$\Rightarrow A = 45^0$$
And
$$\sin B = \frac{1}{\sqrt{2}}$$
$$\Rightarrow B = 45^0$$

$\therefore$cos (A+B)= cos (45°+45°) = cos 90° = 0

16. Find the distance between points (5, -8) and (-7 , -3).

Ans. Given points are (5, -8) and (-7 , -3).

We know,

Distance between points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ Distance between (5, -8) and (-7 , -3) = $\sqrt{(-7-5)^2 + (-3-(-8))^2}$

$$= \sqrt{(-12)^2 + (5)^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169}$$
$$= 13$$

17. Find the co- ordinates on x-axis, of the point which is equidistant from (2,-5) and (-2, 9).

Ans. Let the point on x-axis be (x,0)

Distance between (x,0) and (2,-5) = $\sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$

Distance between (x,0) and (-2,9) = $\sqrt{(x-(-2))^2 + (0-9)^2} = \sqrt{(x+2)^2 + (-9)^2}$

According to question,

$$\sqrt{(x-2)^2+(5)^2}=\sqrt{(x+2)^2+(-9)^2}$$

Squaring both the sides, we get

$$(x-2)^2+(5)^2=(x+2)^2+(-9)^2$$
$$\Rightarrow x^2-4x+4+25=x^2+4x+4+81$$
$$\Rightarrow 8x=25-81$$
$$\Rightarrow 8x=56$$
$$\therefore x=7$$

Therefore, the point is (- 7, 0).

18. The co-ordinate of mid-point of line AB is (2, 4).If the co-ordinate of A is (5, 7), then find the co-ordinate of B.

Ans. Let, the mid-point of AB line be C, having co-ordinates (2,4).

Given, co-ordinate of point A is (5, 7). Let, the point B be (x ,y).

Since C is the mid -point of AB line,

So,

$$(2,4)=\left\{\frac{(5+x)}{2},\frac{(7+y)}{2}\right\}$$

Now,

$$2=\frac{5+x}{2}$$
$$\Rightarrow 4=5+x$$
$$\therefore x=-1$$

And,

$$4=\frac{7+y}{2}$$
$$\Rightarrow 8=7+y$$
$$\therefore y=1$$

Therefore, the co-ordinates of the mid-point of AB line is (-1,1).

19. What are the probability of 'Impossible Event' and 'Sure Event'?

Ans. A sure event is an event which will definitely happen. Hence, its probability is 1.

An impossible event is an event whose occurring possibility is as minimum as possible. Hence, its probability is 0.

20. Write the formula for representing the happening of event A.

Ans. Formula to represent the happening of event A :

$$P(A) = \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$$

21. Write formula to evaluate mean of a classified data.

Ans. Formula to calculate mean of classified data is:

$$\bar{x} = \frac{\sum fx}{n}$$

Where,

$\bar{x}$ = mean

f = frequency of each class

x = mid-interval value of each class

n = total frequency

$\sum fx$ = sum of the product of mid- interval values and their corresponding frequency.

22. What are merits of median? (Any Two)

Ans. Median of a data or distribution is the value of the variable which divides it into two equal parts. Merits of median are:

i. It is easy to calculate and understand.
ii. It is not affected by extreme values and also interdependent of range or dispersion of the data.

Section C

23. Draw the graph of the equations $5x - y = 5$ and $3x - y = 3$. Determine the area of the triangle formed by these lines and the y-axis.

OR

(i) Solve the following pair of linear equation:

$$\frac{x}{a} - \frac{y}{b} = 0 \text{ and } ax + by = a^2 + b^2$$

Ans. The given linear equations are

$$\frac{x}{a} - \frac{y}{b} = 0$$
$$\Rightarrow \frac{x}{a} = \frac{y}{b}$$
$$\Rightarrow x = \frac{ay}{b} ....(i)$$

And,

$$ax + by = a^2 + b^2 ....(ii)$$

Now, putting value of x from (i) in (ii), we get

$$a\left(\frac{ay}{b}\right) + by = a^2 + b^2$$
$$\Rightarrow \frac{a^2 y}{b} + by = a^2 + b^2$$
$$\Rightarrow \frac{a^2 y + b^2 y}{b} = a^2 + b^2$$
$$\Rightarrow \frac{y(a^2 + b^2)}{b} = a^2 + b^2$$
$$\Rightarrow y(a^2 + b^2) = b(a^2 + b^2)$$
$$\therefore y = b$$

And, putting value of y in (i), we get

$$x = \frac{a \times b}{b}$$
$$\therefore x = a$$

(ii) For what value of p and q will the following pair of linear equations have infinitely many solutions?

p(-1)x +3y = 2 ; 6x + (2-q) y = 6

Ans. The given equations are

$$\begin{aligned} &p(-1)x + 3y = 2 \\ &(-p)x + 3y - 2 = 0 \end{aligned} \quad ....(1)$$

$$\begin{aligned} &6x + 2(-q)\, y = 6 \\ &6x - 2qy - 6 = 0 \end{aligned} \quad ....(2)$$

We know, the condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, from equation (1)& (2), we get

$$\frac{-p}{6} = \frac{3}{-2q} = \frac{-2}{-6}$$
$$\frac{-p}{6} = \frac{3}{-2q} = \frac{1}{3}$$

Now,

$$\frac{-p}{6} = \frac{1}{3}$$
$$\Rightarrow -3p = 6$$
$$\therefore\ p = \frac{6}{-3} = -2$$

Also,

$$\frac{3}{-2q} = \frac{1}{3}$$
$$\Rightarrow -2q = 9$$
$$\therefore q = -\frac{9}{2}$$

Hence, the required value of p and q are -2 and $-\frac{9}{2}$ respectively.

24. (i) Prove that parallelogram circumscribing a circle is a rhombus.

Ans. Given ABCD is a parallelogram such that its sides touch a circle with centre O.

∴ AB = CD and AB || CD & AD = BC and AD || BC

Now,

P, Q, R and S are the touching point of both the circle and the parallelogram.

We know that, tangents to a circle from an exterior point are equal in length.

∴ AP = AS [Tangents from point A] ... (1

BP = BQ [Tangents from point B] ... (2)

CR = CQ [Tangents from point C] ... (3)

DR = DS [Tangents from point D] ... (4)

On adding (1), (2), (3) and (4), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = BC + BC [∵ ABCD is a parallelogram , ∴ AB = CD and AD = BC]

2AB = 2BC

AB = BC

Therefore, AB = BC implies

AB = BC = CD = AD

Hence, it is proved that ABCD is a rhombus.

(ii) Two concentric circles are of radii 5cm and 3cm. Find the length of the chord of the larger scale which touches the smaller scale.

Ans. Let the radius of the bigger circle be 'R' and the radius of the smaller circle be 'r'.

Given R=5cm=OB and r=3cm=OD

AB is the chord .

AD=BD and OD⊥AB [∵ radius is perpendicular to a chord and it divides the chord into two equal parts]

∴ΔODB is a right angled triangle

So,

$$OD^2 + BD^2 = OB^2$$
$$\Rightarrow 3^2 + BD^2 = 5^2$$
$$\Rightarrow 9 + BD^2 = 25$$
$$\Rightarrow BD^2 = 16$$
$$\therefore BD = 4cm$$

Since AD=BD=4cm

∴AB =AD+BD

= 4+4 =8cm

Hence, the required length of chord is 8cm.

OR

If x is the area of a right angled ΔABC, in which $\angle ABC$ =90° and BC = b, show that the length of the altitude BN on the hypotenuse AC is $\dfrac{2bx}{\sqrt{b^4 + 4x^2}}$

Ans. Given,

Area of triangle ΔABC = x

$$\Rightarrow \frac{1}{2} \times AB \times BC = x$$

$$\Rightarrow \frac{1}{2} \times AB \times b = x$$

$$\Rightarrow AB = \frac{2x}{b}$$

Now,

By Pythagoras theorem,

$$AC = \sqrt{AB^2 + BC^2}$$

$$\Rightarrow AC = \sqrt{\left(\frac{2x}{b}\right)^2 + b^2}$$

$$\Rightarrow AC = \sqrt{\frac{4x^2}{b^2} + b^2}$$

$$\therefore AC = \sqrt{\frac{4x^2 + b^4}{b^2}} = \frac{\sqrt{4x^2 + b^4}}{b}$$

Again,

Area of triangle = x

$$\Rightarrow \frac{1}{2} \times AC \times BN = x \qquad \text{[Taking base AC and altitude BN]}$$

$$\Rightarrow \frac{1}{2} \times \frac{\sqrt{4x^2 + b^4}}{b} \times BN = x$$

$$\Rightarrow BN = \frac{2x}{\frac{\sqrt{4x^2 + b^4}}{b}}$$

$$\therefore BN = \frac{2bx}{\sqrt{b^4 + 4x^2}}$$

Hence proved.

25. From the top of a 7m high building, the angle of elevation of the top of cable tower is 60°and angle of depression of its foot is 45°. Find the height of the tower.

Ans. In ΔABC,

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{7}{y}$$

$$\therefore y = 7m$$

In ΔAED,

$$\tan 60^0 = \frac{DE}{AE}$$

$$\Rightarrow \sqrt{3} = \frac{x}{y}$$

$$\Rightarrow \sqrt{3} = \frac{x}{7} \quad \text{[From (i)]}$$

$$\therefore x = 7\sqrt{3}$$

Now,

Height of tower = DE +EC

= x + 7

= $7\sqrt{3}+7$

= $7(\sqrt{3}+1)$

Hence, the height of tower is $7(\sqrt{3}+1)$m.

OR

Prove that-

(i) $(\cos ec\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Ans. L.H.S =

$$(\cos ec\theta - \cot\theta)^2$$
$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$
$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$
$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} \qquad [\because \sin^2\theta + \cos\theta^2 = 1]$$
$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$
$$= \frac{1-\cos\theta}{1+\cos\theta}$$
$$= R.H.S$$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

Ans. L.H.S =

$$\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$
$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$
$$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1+\sin A)}$$
$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)} \qquad [\because \sin^2 A + \cos^2 A = 1]$$
$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$
$$= \frac{2}{\cos A}$$
$$= 2\sec A$$
$$= R.H.S$$

26. From a solid cylinder of height 14cm and base diameter 7cm two equal conical holes each of radius 2.1cm and height 4cm are cut-off. Find the volume of remaining solid.

Ans. Given, height of cylinder = 14cm

Height of conical hole= 4cm

Radius of conical hole = 2.1cm

Diameter =7cm

So, radius of cylinder= 3.5

We know,

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (3.5)^2 \times 14$$
$$= 22 \times 3.5 \times 3.5 \times 2$$
$$= 539 cm^3$$

Volume of colonial holes = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4$$
$$= \frac{22 \times 2.1 \times 2.1 \times 4}{21}$$
$$= 18.48 cm^3$$

There are two colonial holes, so

$(2 \times 18.48)\ cm^3 = 36.96\ cm^3$

Therefore, the remaining volume = (539 – 36.96) cm^3 =502.04 cm^3

OR

A right angled triangle, whose sides are 12cm and 5cm, is made to revolve about its hypotenuse. Find the volume of the solid formed.

Ans. When the triangle is made to revolve about hypotenuse so,

Radius of Cone (r) = 5 cm

and, height (h) = 12 cm

We know,

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 12$$
$$= \frac{22 \times 25 \times 12}{21}$$
$$= 314.28 cm^3$$

Therefore, the volume of the solid formed is $314.28\,cm^3$